Search for a Sterile Neutrino in a 3+1 Framework using Wire-Cell Inclusive Charged-Current $\nu_e$ Selection with the BNB and NuMI beamlines in MicroBooNE

The MicroBooNE collaboration
microboone_info@fnal.gov

June 7, 2024
Contents

1 Introduction .......... 3
2 Analysis approach ... 4
3 Analysis Status ..... 9
   3.1 $\nu_\mu$ CC data/MC with updated NuMI flux .......... 9
   3.2 Sensitivity Results ...................................... 11
4 Summary ............... 11
1 INTRODUCTION

While most neutrino oscillation measurements are consistent with the three-neutrino framework (see Ref. [1, 2] among others), the existence of a light eV-scale sterile neutrino has been postulated to explain several experimental anomalies: (1) the observation that calibrated $\nu_e$ sources ($^{51}$Cr for GALLEX [3] and BEST [4], $^{51}$Cr and $^{37}$Ar for SAGE [5]) observed lower rates of $\nu_e$ interactions than expected in the three-neutrino framework, which could be explained by $\nu_e$ disappearance considering light sterile neutrinos; (2) the reactor anti-neutrino anomaly [6], where the observed deficit in the measured $\bar{\nu}_e$ events relative to the expectation based on the recent reactor anti-neutrino flux calculations [7, 8] could be explained by $\bar{\nu}_e$ disappearance considering light sterile neutrinos, although there are recent experimental measurements [9, 10] and improved flux calculations [11, 12] that disfavor this explanation; (3) the Neutrino-4 [13] anomaly, which suggests reactor $\bar{\nu}_e$ oscillation at a few meters; and (4) the anomalous excess of electron-neutrino-like events in LSND [14] and the excess of low-energy electron-like (LEE) events in MiniBooNE [15, 16], which suggest $\nu_e$ appearance from $\nu_\mu$ to $\nu_e$ oscillations as might occur considering light sterile neutrinos. However, there are significant challenges in explaining all available experimental results with a sterile neutrino oscillation model in a global fit [17]. Nevertheless, it is important to clarify these experimental anomalies. Moreover, a sterile neutrino, if discovered, would have a profound impact on not only particle physics but also astrophysics and cosmology such as in large-scale structure formation [18] and leptogenesis [19].

The recent distinct and complementary low-energy excess searches at MicroBooNE [20, 21, 22, 23], which aim to provide a definite check on the MiniBooNE LEE, show that “the results are found to be consistent with the nominal $\nu_e$ rate expectations from the Booster Neutrino Beam (BNB) [24] and no excess of $\nu_e$ events is observed”, assuming a simple LEE template unfolded from the MiniBooNE excess. While these results suggest the MiniBooNE LEE has a non-$\nu_e$ origin, the current results may still be compatible with the hypothesis of a light sterile neutrino suggested by remaining experimental anomalies (GALLEX [3], BEST [4], SAGE [5], Neutrino-4 [13], and LSND [14]).

In order to fully evaluate the possible existence of sterile neutrinos using the MicroBooNE data, a 3+1 (three flavors of standard model neutrinos + one flavor of sterile neutrino) neutrino oscillation analysis was carried out previously, using only BNB data [25]. The high-performance neutrino selection and well-understood systematic uncertainties of the $\nu_e$ and $\nu_\mu$ event rate predictions in the recent MicroBooNE LEE analysis using Wire-Cell reconstruction [23] was used, and the 3+1 oscillation analysis results for several different scenarios considering the oscillation effects from (1) $\nu_e$ appearance only ($\nu_\mu$ to $\nu_e$ oscillation), (2) $\nu_e$ disappearance only, and (3) $\nu_e$ appearance + $\nu_e$ and $\nu_\mu$ disappearance was presented. The sensitivity of our first result is impacted by a potential cancellation of $\nu_e$ appearance and disappearance. This degeneracy in the oscillation signature is primarily impacted by the relative content of $\nu_\mu$ and $\nu_e$ in the beam. The addition of data from the Neutrinos at the Main Injector (NuMI) beamline (a second neutrino beam where off-axis neutrinos incidently illuminate the MicroBooNE detector) in a joint oscillation measurement can mitigate
this degeneracy enhancing the analysis’ sensitivity thanks to the different overall $\nu_\mu$ -to-$\nu_e$ ratio of the two beams.

In this note, we show the sensitivity improvement by adding in data collected from the NuMI beam during 2015-18 in the same analysis framework. In addition, a validation of the NuMI flux prediction at MicroBooNE [26] is performed.

2 Analysis Approach

In this analysis, we consider the 3+1 neutrino framework, which is the simplest extension of the standard 3$\nu$ framework with the addition of a non-standard massive (sterile) neutrino at the eV scale. The flavor and mass eigenstates are connected by a $4 \times 4$ mixing matrix

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix}.
\]

The oscillation probability from $\alpha$-flavor to $\beta$-flavor type neutrino in vacuum can be expressed as

\[
P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\beta i}U_{\alpha j}^*)\sin^2 \Delta_{ij} + 2 \sum_{i>j} \text{Im}(U_{\beta i}U_{\alpha j}^*)\sin 2\Delta_{ij},
\]

where $\alpha, \beta = e, \mu, \tau, s$, and $i, j = 1, 2, 3, 4$. $\Delta_{ij}$ stands for

\[
\Delta_{ij} = \frac{\Delta m^2_{ij} L}{4E} = 1.267 \left( \frac{\Delta m^2_{ij}}{\text{MeV}^2} \right) \left( \frac{L}{E} \right) \left( \frac{E}{m} \right)
\]

where $\Delta m^2_{ij} = m_i^2 - m_j^2$ is the mass-squared difference between the neutrino mass eigenstates $\nu_i$ and $\nu_j$.

Since $m_4$ is at eV-scale ($m_4 \gg m_3, m_2, m_1$) and the MicroBooNE experiment has a short baseline, there is effectively only one mass-squared difference $\Delta m^2_{41}$ appearing in the oscillation formula. The three main oscillation channels: $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\mu$, and $\nu_\mu \to \nu_e$, have the following oscillation probabilities:

\[
P_{\nu_e \to \nu_e} = 1 - 4(1 - |U_{e4}|^2)|U_{e4}|^2\sin^2 \Delta_{41},
\]

\[
P_{\nu_\mu \to \nu_\mu} = 1 - 4(1 - |U_{\mu4}|^2)|U_{\mu4}|^2\sin^2 \Delta_{41},
\]

\[
P_{\nu_\mu \to \nu_e} = 4|U_{\mu4}|^2|U_{e4}|^2\sin^2 \Delta_{41}.
\]
The matrix element terms are often replaced with effective mixing angles

\[
\sin^2 2\theta_{ee} = 4(1 - |U_{e4}|^2)|U_{e4}|^2, \tag{7}
\]

\[
\sin^2 2\theta_{\mu\mu} = 4(1 - |U_{\mu4}|^2)|U_{\mu4}|^2, \tag{8}
\]

\[
\sin^2 2\theta_{\mu e} = 4|U_{\mu4}|^2|U_{e4}|^2. \tag{9}
\]

The \(\nu_e \rightarrow \nu_s\) and \(\nu_\mu \rightarrow \nu_s\) oscillation probabilities, which are important for the neutral-current interaction channel, are

\[
P_{\nu_e \rightarrow \nu_s} = 4|U_{e4}|^2|U_{s4}|^2 \sin^2 \Delta_{41}, \tag{10}
\]

\[
P_{\nu_\mu \rightarrow \nu_s} = 4|U_{\mu4}|^2|U_{s4}|^2 \sin^2 \Delta_{41}. \tag{11}
\]

Similarly, we can replace the matrix terms with effective mixing angles

\[
\sin^2 2\theta_{es} = 4|U_{e4}|^2|U_{s4}|^2, \tag{12}
\]

\[
\sin^2 2\theta_{\mu s} = 4|U_{\mu4}|^2|U_{s4}|^2. \tag{13}
\]

Here, the \(4 \times 4\) mixing matrix can be parameterized as below ensuring the unitarity of the mixing matrix \([27]\),

\[
U = R^{34}(\theta_{34}, \delta_{34})R^{24}(\theta_{24}, \delta_{24})R^{14}(\theta_{14}, 0)R^{23}(\theta_{23}, 0)R^{13}(\theta_{13}, \delta_{13})R^{12}(\theta_{12}, 0) \tag{14}
\]

where \(R_{ij}(\theta_{ij}, \delta_{ij})\) denotes a counterclockwise rotation in the complex \(ij\)-plane through a mixing angle \(\theta_{ij}\) and a \(CP\) phase \(\delta_{ij}\). Table 1 shows the connections between the mixing angles and the mixing matrix terms.

**Table 1:** 3+1 sterile neutrino mixing parameters and the effective angles.

| \(\sin^2 2\theta_{ee}\)          | \(\sin^2 2\theta_{14}\)          | \(= 4(1 - |U_{e4}|^2)|U_{e4}|^2\) |
|---------------------------------|----------------------------------|----------------------------------|
| \(\sin^2 2\theta_{\mu\mu}\)    | \(4\cos^2 \theta_{14}\sin^2 \theta_{24}(1 - \cos^2 \theta_{14}\sin^2 \theta_{24})\) | \(= 4(1 - |U_{\mu4}|^2)|U_{\mu4}|^2\) |
| \(\sin^2 2\theta_{\mu e}\)     | \(\sin^2 2\theta_{14}\sin^2 \theta_{24}\) | \(= 4|U_{\mu4}|^2|U_{e4}|^2\)    |
| \(\sin^2 2\theta_{es}\)       | \(\sin^2 2\theta_{14}\cos^2 \theta_{24}\cos^2 \theta_{34}\) | \(= 4|U_{e4}|^2|U_{s4}|^2\)    |
| \(\sin^2 2\theta_{\mu s}\)    | \(\cos^4 \theta_{14}\sin^2 2\theta_{24}\cos^2 \theta_{34}\) | \(= 4|U_{\mu4}|^2|U_{s4}|^2\)    |

It should be noted that the \(\nu_e\) disappearance oscillation effect can cancel the \(\nu_e\) oscillation effect in the observed \(\nu_e\) CC events, resulting in a degeneracy in the estimated oscillation parameters. This cancellation is expressed in in the equation
\[ N_{\nu_e} = N_{\text{intrinsic} \, \nu_e} \cdot P_{\nu_e \rightarrow \nu_e} + N_{\text{intrinsic} \, \nu_\mu} \cdot P_{\nu_\mu \rightarrow \nu_e} \]
\[ = N_{\text{intrinsic} \, \nu_e} \cdot \left[ 1 + (R_{\nu_\mu/\nu_e} \cdot \sin^2 \theta_{24} - 1) \cdot \sin^2 2\theta_{14} \cdot \sin^2 \Delta_{41} \right] , \]

where \( R_{\nu_\mu/\nu_e} \) is the ratio of intrinsic \( \nu_\mu \) and \( \nu_e \) events in the beam as a function of true neutrino energy. In the case of the BNB, the degeneracy of \( \sin^2 \theta_{24} \) and \( \sin^2 2\theta_{14} \) happens when \( \sin^2 \theta_{24} \approx 0.005 \) given \( R_{\nu_\mu/\nu_e} \approx 185 \). This degeneracy is mitigated by adding data from NuMI beamline, where \( R_{\nu_\mu/\nu_e} \approx 21 \). Figure 1 shows that the intrinsic flux and \( \nu_\mu \) to \( \nu_e \) ratio in NuMI is quite different from that in BNB across the relevant range of energies.

\[ F_{\nu_e} = \frac{N_{\nu_e}}{N_{\text{intrinsic} \, \nu_e} \cdot \left[ 1 + (R_{\nu_\mu/\nu_e} \cdot \sin^2 \theta_{24} - 1) \cdot \sin^2 2\theta_{14} \cdot \sin^2 \Delta_{41} \right]} , \]

For NuMI, FHC (forward horn current, solid lines) and RHC (reverse horn current, dashed lines) predictions are shown together.

Figure 2 illustrates the predicted energy spectra of the BNB and NuMI \( \nu_e \) CC FC channels at different values of the oscillation parameters\(^1\): (a) no oscillation effect (black solid line), (b) both disappearance and appearance oscillation effects with \( (\Delta m^2_{14} = 7.3 \text{ eV}^2, \sin^2 2\theta_{14} = 0.36, \sin^2 \theta_{24} = 0.01) \) (blue solid line), (c) both disappearance and appearance oscillation effects with \( (\Delta m^2_{14} = 7.3 \text{ eV}^2, \sin^2 2\theta_{14} = 0.36, \sin^2 \theta_{24} = 0.005) \) (red solid line), and (d) both disappearance and appearance oscillation effects with \( (\Delta m^2_{14} = 7.3 \text{ eV}^2, \sin^2 2\theta_{14} = 0.72, \sin^2 \theta_{24} = 0.005) \) (red dashed line).

In BNB, the scenario (b) show obvious oscillation effects. The scenarios (c) and (d) show weak

\(^1\)Note that \( \nu_e \) CC channels contain both \( \nu_e \) and \( \bar{\nu}_e \) components. The \( \bar{\nu}_e \) rate is negligible in BNB \( \nu_e \) CC selection, whereas in the NuMI \( \nu_e \) CC selection, it constitutes 1/5 of the \( \nu_e \).
oscillation effect below 1500 MeV, which is because of the cancellation between $\nu_e$ disappearance and appearance, especially when $\sin^2\theta_{24}$ approaches 0.005. This shows the impact of the degeneracy in the BNB $\nu_e$ dataset.

In NuMI, this degeneracy is broken due to the different $R_{\nu_\mu/\nu_e}$, so there's no such degeneracy with $\sin^2\theta_{24} = 0.005$. One can see that the scenario (c) and (d) result in very different spectra, causing no degeneracy in $\sin^2\theta_{14}$ for NuMI. Also, in both cases, the impact of oscillations on the energy spectrum is clearly visible.

![Energy spectra and event rate ratio](Image)

**Figure 2**: Energy spectra and event rate ratio of oscillation/no oscillation for the BNB and NuMI $\nu_e$ CC FC channel at different values of the oscillation parameters.

**Figure 3** shows a similar study, but with different oscillation parameters. Here we pick $\Delta m_{14}^2 = 1.2 \text{ eV}^2$, $\sin^22\theta_{\mu e} = 0.003$, and compared different predicted energy spectrum of the BNB and NuMI $\nu_e$ CC FC channels: (a) no oscillation effect (black solid line), (b) both disappearance and appearance oscillation effects with $(\Delta m_{14}^2 = 1.2 \text{ eV}^2$, $\sin^22\theta_{\mu e} = 0.003$, $\sin^2\theta_{24} = 0.018$) (red dashed line), and (c) both disappearance and appearance oscillation effects with $(\Delta m_{14}^2 = 1.2 \text{ eV}^2$, $\sin^22\theta_{\mu e} = 0.003$, $\sin^2\theta_{24} = 0.0045$) (greed solid line). Here, $\sin^2\theta_{24}$ values were chosen based on preferred value with either the combined BNB and NuMI Asimov dataset, or the BNB-only data.
Similar to the study in Fig. 2, case (b) shows clear oscillation effect in BNB, where case (c) with weak oscillation effect with $\sin^2\theta_{24}$ close to the degeneracy point $\sin^2\theta_{24} = 0.005$. In NuMI, this degeneracy is broken as case (c) shows the strong oscillation with no degeneracy in this oscillation parameter values.

![BNB reconstructed neutrino energy spectrum](image)

![NuMI reconstructed neutrino energy spectrum](image)

![BNB event rate ratio](image)

![NuMI event rate ratio](image)

**Figure 3:** Energy spectra and event rate ratio of oscillation/no oscillation for the BNB and NuMI $\nu_e$ CC FC channel at different values of the oscillation parameters.


3 Analysis Status

3.1 $\nu_\mu$ CC data/MC with updated NuMI flux

As described in Ref. [26], the NuMI flux prediction at MicroBooNE was recently modified to include several updates:

- Shielding block geometry update
- Geant4 version update from v4.9.2 to v4.10.4
- Updated PPFX implementation accounting for the underlying changes in the simulation

The MicroBooNE collaboration has developed a reweighting scheme in order to reflect these changes in the NuMI flux prediction, with more details in Ref. [26]. With this new NuMI flux prediction, the overall uncertainty on the NuMI flux is about 20%. We have checked the validity of the new flux prediction and its uncertainty by looking at the sideband samples, $\nu_\mu$ CC fully-contained (FC) and partially-contained (PC) events.

Figure 4 shows the reconstructed neutrino energy spectrum of NuMI Run1-3 $\nu_\mu$ CC FC and PC samples. We observe an underprediction of Monte-Carlo (MC), especially at lower energies, with about 26% lower than data. However, the offset is consistent at the 1 \( \sigma \) level within the systematics.

In order to further validate the newly updated NuMI flux prediction and its uncertainty, we have performed a conditional constraint study using the BNB $\nu_\mu$ CC FC sample. The conditional covariance matrix formalism [28] was used for the study, where we derive the conditional mean and conditional variance of the prediction of target channel (NuMI $\nu_\mu$ CC FC in Fig. 6a or NuMI $\nu_\mu$ CC PC sample in Fig. 6b), given the constraints from the measurement of constraining channels (BNB...
νμ CC FC in Fig. 6a or NuMI νμ CC FC sample in Fig. 6b). This allows more information about the compatibility between the model and data.

Figure 5 shows the energy spectrum of both the BNB and NuMI νμ CC FC samples with the total unconstrained systematic uncertainty, and Fig. 6 shows the NuMI νμ CC FC constrained by the BNB νμ CC FC sample and the NuMI νμ CC PC sample constrained by the NuMI νμ CC FC sample. When constraining the NuMI νμ CC FC prediction using the BNB νμ CC FC observed data, the correlated cross-section and detector systematic uncertainties are strongly suppressed. The flux systematics are unchanged by the constraint since they are treated as uncorrelated. Hence, the remaining uncertainty is dominated by the flux uncertainty. The data and constrained prediction show good agreement, within flux-dominated systematics, demonstrating that the updated flux prediction agrees well with the data well within the uncertainty. This can further be checked by constraining the NuMI νμ CC PC by NuMI νμ CC FC sample, where the conditional constraints suppress all the systematic errors leading to a significantly reduced uncertainty. The data and constrained prediction agrees very well within the remaining uncertainty with $\chi^2/ndf = 13.76/25$.

![Figure 5: Reconstructed energy spectrum of BNB Run1-3 and NuMI Run1-3 νμ CC FC sample. The red band represents the total systematic uncertainty. The bottom sub-panel presents the data-to-prediction ratio as well as the full systematic uncertainty of MC prediction.](image)

In summary, we have observed that our updated NuMI flux prediction agrees well with νμ data observations within systematic uncertainties, validating the usage of this flux for this oscillation analysis.
3.2 Sensitivity Results

We have performed a sensitivity study in $\nu_e$ appearance and $\nu_e$ disappearance channels, using both the BNB and NuMI datasets. The sensitivity shown here is median sensitivity calculated using the frequentist CLs method [29].

Figure 7 shows the $\nu_e$ appearance channel, with the full 3+1 oscillation sensitivity after profiling over the mixing angle $\sin^2 \theta_{24}$, using the frequentist CLs method with 2000 pseudo experiments. Compared to the BNB-only result, the BNB and NuMI combined result improves the sensitivity significantly, as a result of breaking the degeneracy of oscillation parameters. With BNB and NuMI combined, the sensitivity covers the majority of the LSND 90% allowed region.

Figure 8 shows the $\nu_e$ disappearance channel. Similar to the $\nu_e$ appearance channel, the method of frequentist CLs was used to calculate profiled sensitivity for BNB-only and BNB/NuMI combined scenarios. By combining the BNB and NuMI, we recover the loss of sensitivity attributed to the parameter degeneracy and obtain sensitivity that covers much of the Gallium result.

4 SUMMARY

In this note, we present the 3+1 sterile neutrino oscillation analysis sensitivities using both the BNB and NuMI beams, built upon the previous result of BNB-only 3+1 oscillation analysis [25]. With flux model updates in MicroBooNE’s simulation of the NuMI flux, we report the sensitivity results.
Figure 7: MicroBooNE 95% confidence level frequentist CLs limits in the $\Delta m^2_{41}$ vs. $\sin^2 2\theta_{\mu\nu}$ parameter space. The LSND 90% and 95% CL allowed regions [14] are shown in shaded areas. The black dashed and solid lines are BNB-only sensitivity and data exclusion result, respectively, from previous result [25]. The blue dashed curve represents the MicroBooNE 95% CLs median sensitivity (2D profiling by minimizing over $\sin^2 2\theta_{24}$) in the full 3+1 oscillation scenario (both $\nu_e$ appearance and $\nu_e$ & $\nu_{\mu}$ disappearance) using BNB Run1-3 and NuMI Run1-3 combined. The $1\sigma$ and $2\sigma$ bands around the median as shown as green and yellow shaded areas respectively.

Figure 8: MicroBooNE 95% confidence level frequentist CLs limits in the $\Delta m^2_{41}$ vs. $\sin^2 2\theta_{ee}$ parameter space. The GALLEX+SAGE+BEST [4] and Neutrino-4 [13] $2\sigma$ allowed regions are shown in shaded areas. The black dashed and solid lines are BNB-only sensitivity and data exclusion result, respectively, from previous result [25]. The blue dashed curve represents the MicroBooNE 95% CLs median sensitivity (2D profiling by minimizing over $\sin^2 2\theta_{24}$) in the full 3+1 oscillation scenario (both $\nu_e$ appearance and $\nu_e$ & $\nu_{\mu}$ disappearance) using BNB Run1-3 and NuMI Run1-3 combined. The $1\sigma$ and $2\sigma$ bands around the median as shown as green and yellow shaded areas respectively.
for the full oscillation case that considers both the $\nu_e$ appearance and disappearance. The results show that adding the NuMI data to the analysis gains significant improvements in both channels, becoming sensitive to LSND and Gallium allowed regions. These gains follow from the different $\nu_\mu$-to-$\nu_e$ ratios in the NuMI and BNB beams, which significantly reduce the oscillation parameter degeneracy.

REFERENCES


